LENSES AND MIRRORS

PART 1.

ROGERS

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AUTHOR AND PUBLISHER

NEW CORRESPONDENCE COURSES IN THE

Theory and Practice

OF

OPTOMETRY

SPECIAL COURSE

No. 2

Lenses and Mirrors

PART I.

BY

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FOREWORD

This booklet is one of a series of three on Lenses and Mirrors. The purpose of the series is to give the student of optometry a more thorough acquaintance with these agents than he can obtain from the study of those general texts, with which all are familiar, including Hartridge, Thorington, Alger and many others.

It is a special series in that it gets below the surface of an ordinary description of a lens or mirror, and establishes those primary facts and principles that are necessary to a full understanding of what can be accomplished optically with them. For the sake of setting these special facts and principles before students, much of the ordinary descriptive matter that applies to lenses is omitted, it being assumed that the student is well acquainted with them.

A lens or a mirror varies in action slightly from its principal axis out toward the periphery. To differentiate these differences would consume much space and be of no practical value. We confine our attention to the axial areas—that is, to a limited space immediately surrounding the poles. Even this limited area has a different action at different distances from the principal axis, but such differences are treated as negligible factors, since the action itself is due to these differences.

In this booklet the central thickness of the lens is treated as nonexistent, or the actions at the two surfaces are treated as though they were one action, and at the plane of the optical center. It is only when such thickness is considerable that it exercises a perceptible modifying effect upon the whole lens action. Of course, even in the thinnest lenses, it is a modifying factor; but in lenses such as those employed in optometry its influence is exceedingly slight.

In the second booklet of the series, which treats of modified lens values, as well as in this, lenses are treated as though they had no central thickness, or as though they were "infinitely thin." But in the third booklet these neglected factors will all be taken into account, although even then it will be the central areas only of a lens that will be given attention. If one is acquainted with just what this area of a lens does, he has the essence of its purpose and action, and he may cut out the periphery with an iris diaphragm.

The sort of information that is contained in the series of booklets is not wanting in practical value and utility. New questions are constantly coming up in which the most precise information is necessary to work out a constructive problem. The author is having these questions submitted to him from time to time. Among these questions are the following:

1.—What mirror value is obtained from silvering one surface of a given lens?

2.—How can the light from a small area of molton metal be concentrated upon a limited space?

3.—How may under-water lenses be ground to correct the eyes and give emmetropic vision?

4.—Problems that relate to war instruments, periscopes, range finders, etc.

To even undertake any of these problems one must have a fuller knowledge of how a lens or mirror acts upon light from any distance, and just how the images are formed. We analyze these actions dioptrically, by the wave theory, and also by the focal-length system. To most optometrists the latter method will be new. But there are many optometrists who can go through the procedure of fitting the eyes of a patient with lenses perfectly who yet know all too little about the means they employ for that purpose.

The time is coming when the optometrist's business will not be confined to the mere fitting of the eyes. He will be appealed to by many people who want special information on some point connected, in a more remote way, with optics or optometry. The meeting of these demands will tend to raise the standing of the optometrist in the estimation of those who require the special service. On the other hand, if he is unable to give the information, it must reflect upon the proficiency of one to even do the simple work of fitting the eyes.

This booklet, it will be seen, is a part of our New Correspondence Courses in the Theory and Practice of Optometry.

GEO. A. ROGERS.

Chicago, July 15, 1918.

I

LENSES AND MIRRORS.

A lens is a thin disc of glass with polished geometric surfaces. In optometry it is placed before the eye for the purpose of modifying the light that passes through it into the eye.

The value or power of the lens is expressed in diopters, which depends upon the form given its geometric surfaces, and the optical density of the glass of which it is made. By the term "geometric surfaces" either plane, spherical or cylindrical surfaces are included; and the spherical or cylindrical surfaces may be convex or concave. As a lens has two such surfaces, both come under the same rule.

A mirror is made of opaque material, usually some metal that is susceptible to a high polish. It has but one surface of action, and that may be plane, spherical or cylindrical. Mirrors are usually plane or spherical, and they may be convex or concave. The most usual form of mirrors employed for optometric purposes are either plane or concave.

The common property of lenses and mirrors that make them so valuable is the fact that, although they may change the form of the light they act upon, they preserve the identity of the waves. The waves of light emerge from lenses or are reflected back by mirrors not as from a new source, but as the same waves they were at incidence. In discussing these agents, the mirror is usually considered first, as its principle is simpler.

The Plane Mirror.

A plane mirror is a polished plane metallic surface that reflects the waves of light in the same form they come to it. Commercially it is made by spreading a metallic amalgam over one surface of a plate of glass, which imposes the glass plate between the object and the reflecting surface. But optical mirrors are now

provided in which the metallic surface is on the front surface of a clay body.

Figure 1 represents the reflection of light by a plane mirror of the latter kind. From every point of the object **f** a pencil of light proceeds to the mirror and is reflected by it. At incidence the waves face

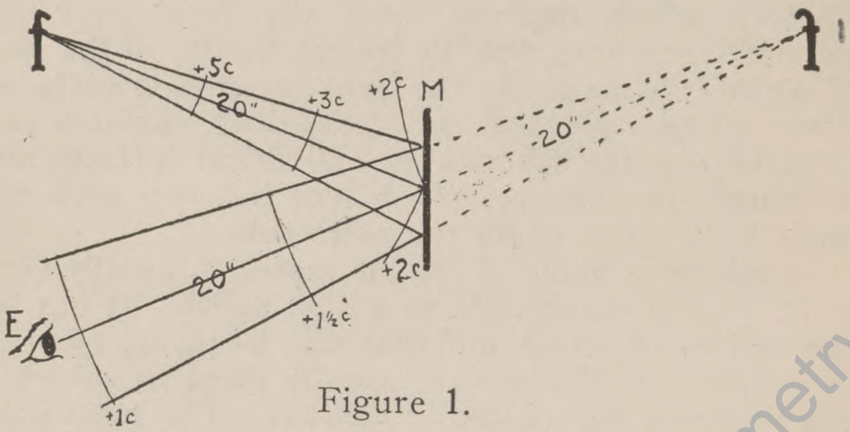

Figure 1.

the mirror, but the reflected waves face in the opposite direction. Hence, to the eye at **E** the object appears at **f′** an equal distance back of the mirror, and reversed from right to left.

What the mirror does to incident light from **f** may be stated as follows:

1. It reverses the direction of propagation.
2. It reverses the wave frontages.
3. It creates a virtual image of **f** at **f′**.

While the law of incidence and reflection is operative, it is of much less importance than the effects stated above, which are usually omitted from the statement of a mirror action.

Light waves come back from a plane mirror in much the same manner as sound waves from a cliff or wood that gives back an echo. When you call "hel-lo" to the cliff the echo comes back "hel-lo," showing that those vibrations that are first to reach

the cliff come back first, or that their order is reversed. But otherwise the waves are unmodified by a plane mirror, except that some of their force is absorbed.

If the object **f** is 20″ =½ meter from the mirror, incident waves at the mirror are +2c in metric curvature. As the plane mirror doesn't modify them in this respect, the reflected waves are also +2c when they leave it, or they appear to come from **f′**. The virtual image, **f′**, is the same distance back of the mirror as **f** is forward of it. But notice, **f′** is reversed from right to left.

If the metallic reflector shown in Figure 1 were not polished, the incident waves of light reaching it would be entirely broken up in organization, the same as a soup bubble striking an obstacle. They would then merely illuminate the surface, but from these illuminated points a new system of spherical waves would be generated as though they were so many luminous points. The opaque rough reflector is the greatest of all means for distributing light.

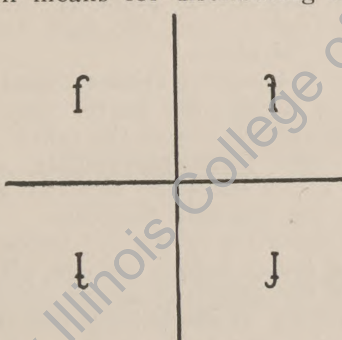

Figure 2.

An object placed above a plane mirror would be reversed in the virtual image in the same manner. But, on account of the direction of incident light from

it, it would be reversed vertically. It takes two reversals together, one horizontal and one vertical, to constitute an inversion. Of the four f's in Figure 2, the original being **f,** the lower right hand one is an inversion.

Optometric Uses.

Plane mirrors are employed in optometry for two principal purposes, (1) as the mirror in retinoscopes, and (2) to double the testing space. In retinoscopes the mirror is small and usually round, and either has a central perforation, or the amalgam is removed from a small central area. For doubling the testing space, a mirror of about 8x10 inches is large enough, though a larger one may be used.

In retinoscopy light is reflected into the eye, forming a small illuminated area upon the retina. Light from this area passes back through the pupil, and is admitted to the observer's eye through the small central perforation in the mirror. The lighted area on the retina, as seen in the pupil by the observer, is known as the "reflex," and the dark areas that surround it as the "shadows."

The mirror is mounted upon a round opaque disc. By means of a handle the mirror may be tilted at different angles, which causes the reflex to move, and the observer sees these movements, and can tell whether they are in the same direction as the mirror is tilted or in the opposite direction. This tells him the refractive condition of the eye tested in this way, and by placing lenses before the eye, all motion may be neutralized, revealing the amount of the error.

It is the tilting of the mirror that brings about these movements. By changing the angle of the mirror to incident light, double that change is imparted to the direction of the reflected light, thus providing a means of making the reflex move rapidly. It gives

the operator and observer a means of controlling the actual movements of the reflex, but how it appears to move depends also upon the refractive condition of the eye being tested in this way.

Doubling Testing Space.

The full distance of 6 meters or 20 feet is often hard to obtain in modern offices, so that one has to make a better use of the space he has. By placing a plane mirror 10 ft. from the test charts, which consists of letters and figures horizontally reversed, the patient is able to see a virtual image of the test chart in the mirror, with the letters reversed back to their normal appearance, and at 20 feet.

As this utilization of a simple optical principle may be made a considerable rent saver, we are illustrating the scheme by which this method is applied in Figure 3. The mirror should be on a level with the

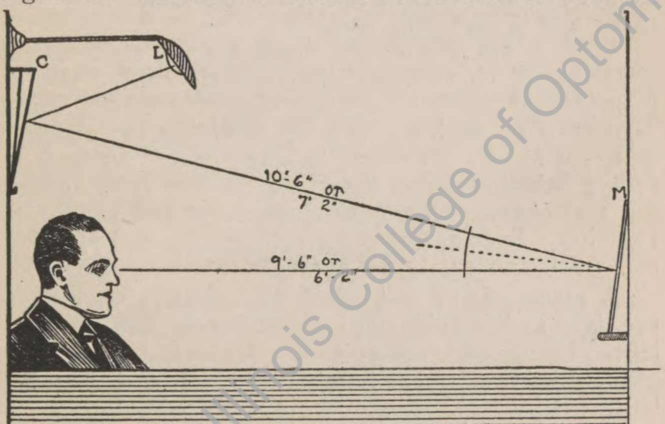

Figure 3.

eyes of the patient, while the test chart, properly illuminated, is best placed above the patient's head. To

square the virtual image to the line of vision, both test chart and mirror should be tilted as shown in the figure.

By this arrangement the patient is unable to see his own or the operator's image in the mirror, but only sees the virtual image of the test type. It is apparently straight in front of him and at right angles to his line of vision. The object of the latter is not to show an astigmatic chart at an angle, which makes the lines appear of unequal distinctness to a normal eye.

The usual testing distance is 6 meters, which is regarded as infinity. But it represents a divergence of the rays of $^1/_6$ diopter, which is not represented by any of the standard fractional values of lenses. A distance of 4 meters answers the purpose quite as well, and the fractional equivalent of this distance is .25 D., for which there is a lens equivalent in all trial cases, and in surfacing tools.

With a reverse chart, made up for this testing distance, all characters being exactly of 2/3 the dimensions of those in 6-meter charts, a testing space of about 7 ft., with a mirror for doubling the distance, makes a most convenient testing space. Aside from saving valuable space, there is a distinct value in having test charts, illumination, operator and patient all together. Special lights for retinoscopy and Maddox-rod work can easily be arranged at the same point.

A plane mirror is neutral dioptrically. It reflects the light as it receives it, and corresponds to a neutral lens. Its dioptric value is 0. It makes no images other than the virtual image described, and its focal-length is infinite. However, it is an exceedingly valuable optometric agent for the purposes that have been stated. A concave mirror, though often used in the retinoscope, would be of no value as a testing space reflector.

II.

SPHERICAL MIRRORS.

Spherical mirrors, the same as plane ones, reverse the direction of light propagation and the wave frontages, and they may produce either real or virtual images of objects placed before them. The images produced by a convex spherical mirror are necessarily virtual and of less size than the object. A concave mirror produces, according to the position of the object, real images of greater or less size than the object, or virtual images of greater size.

The metric curvature of a mirror is the reciprocal of its radius in meters. Therefore, if its radius of curvature is ½ meter or 20″, its metric curvature is the reciprocal of ½ = 2c. If convex, this curvature is +2c, if concave —2c. This nomenclature has reference to the form of the mirror only. It is a nomenclature that applies also to the surfaces of lenses and to the fronts of light waves. Convexity is positive or +, concavity is negative or —, in form. The metric curvature of all are the reciprocal of their radii in meters.

Convex Mirror.

As convex mirrors are of limited value, they may be dealt with first. Ordinarily an optic agent impresses the opposite of its own form upon the waves of light; but, since mirrors reverse the frontages of the waves, they impress their own kind of curvature. That is, convex mirrors impress convexity on the waves, concave mirrors impress concavity. But the action of a spherical mirror is to impress double its own curvature upon the waves of light it reflects.

That is, in receiving the waves it impresses once its curvature, but in reflecting them it acts for the same amount once more.

Figure 4 represents the action of a convex mirror of +2c on waves of light from a point 1 meter distant. Incident waves at the mirror from the point are +1c. The mirror impresses +4c upon them, so that a curvature of +5c is given to the reflected waves, which focuses them at 8″ posterior to the mirror. An object at **A** would therefore have a virtual image at **B,** erect and 1/5 diameters of the object.

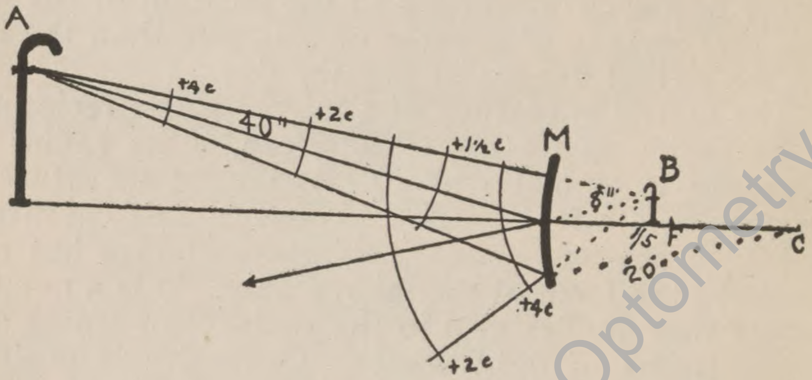

Figure 4.

But the point **B** is not the principal focus of the mirror, for it is not a focal-length of the mirror from it. The focal-length of any spherical mirror is ½ its radius. In the above case, since the mirror has a radius of ½ meter or 20″, it has a focal-length of ½ of 20″ or 10″, and the principal focus is therefore 10″ posterior to the mirror, or at **F.** This is the point at which parallel incident rays of light will be focused by the mirror.

To calculate the position of the image, and from it its size, it is only necessary to determine first the incident curvature of these waves, and then to add

twice the metric curvature of the mirror to it. If, for instance, the object point were at 10″, the incident waves would be +4c, and +4c added to this would make the reflected waves +8c. Their focus would then be 5″ posterior to the mirror. As the relative distances of object and image are 10″ and 5″, the image is ½ the size of the object, and is a virtual image.

A convex mirror scatters or dissipates the light it reflects and produces only small virtual images. It is therefore of little if any optometric value. It is possible that a use for this kind of agents may be discovered, and it is important to understand the principle of it whether it is practically usable or not.

Concave Mirrors.

Concave spherical mirrors are important as optic agents. Their action is to concentrate light, and they produce real images varying from being much smaller, equal to and much larger than the object, depending upon the distance of the object from the mirror. They also produce enlarged virtual images if the object is nearer to the mirror than one focal-length.

Figure 5 represents the action of a concave mirror, radius ½ meter, on light from an object 20″ forward of the mirror. The metric curvature of the mirror

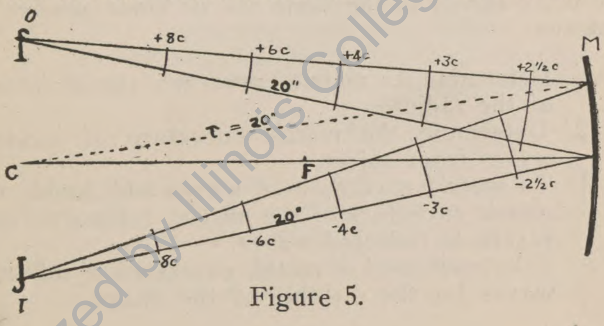

Figure 5.

is —2c, but it impresses —4c on all waves of light it reflects. As the object is at ½ meter, incident waves are +2c, and an impression of —4c gives them a curvature of +2c —4c = —2c. They are therefore focused at 20" forward of the mirror. As object and image are at the same distance from the mirror they are equal in size. The image is real and may be displayed upon a screen.

If the above object were moved to a distance of 40" from the mirror, incident waves from it would be +1c, and the reflected waves for such position would be —3c. This would focus them at 1/3 meter from the mirror, and the image would again be real and 1/3 the dimensions of the object. The relative sizes of object and image are the same as their relative distances from the mirror, or I : O :: Id : Od.

To produce a real image of the same size as the object, the object must be at a distance equal to the radius from the mirror, in which case the image will be at the same distance, and real. To enlarge the image, the object must be at a distance less than the radius but greater than one half of it from the mirror. If the object is nearer than one half a radius, the image will be virtual and back of the mirror.

The method of calculating spherical mirrors by the wave theory is the same for all kinds, convex or concave.

1. Determine its metric curvature, the reciprocal of the radius.
2. Determine the metric curvature of incident waves from object.
3. To metric curvature of waves add double the metric curvature of the mirror, for metric curvature of reflected waves.
4. Take reciprocal of metric curvature of reflected waves for the distance of the image.

5. Compare distance of object with distance of image for sizes.
6. Minus or concave waves make real images; plus or convex waves make virtual images.

Focal-Length System.

By the focal-length system, the principal focus of the mirror is made the datum for measuring distances; and distances of both object and image are expressed in focal-lengths of the mirror from such principal focus. A spherical mirror has but one principal focus. It will be designated **F** for any mirror. It is always at a distance of one focal-length of the mirror from the mirror. For convex mirrors it has a position posterior to the mirror; for concave mirrors it is forward of the mirror. It is the point at which parallel rays are focused.

Certain elementary principles are to be observed in calculating the action of mirrors by this system. These are as follows:

1. Objects and image are always in the same direction from **F.**
2. Whatever the distance of the object from **F,** measured in focal-lengths, the image is the reciprocal of that distance.
3. These respective distances, object or image, are the ratio of image to object or object to image.

In Convex Mirror.

This method is illustrated in Figure 6. **M** is a convex mirror of 12″ radius. Its principal focus is therefore ½ of 12 = 6″ posterior to the mirror, or at **F.** If the object, **O,** is 12″ anterior to the mirror, it is 18″ = 3 focal-lengths of the mirror anterior to

F. Hence, the image is 1/3 of a focal-length, or 2″, anterior to **F,** and is 1/3 the size of the object. As it is back of the mirror it is virtual and therefore erect.

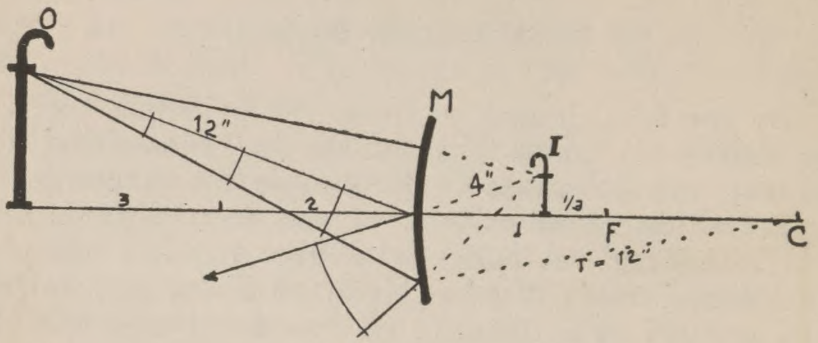

Figure 6.

At a distance of 6″ from the mirror, the object would be 2 focal-lengths of the mirror from **F,** and the image would be ½ a focal-length in the same direction, and ½ the size of the object, but a virtual image as before. This would place it 3″ forward of **F** and 3″ back of the mirror. If the object were moved to 30″ from the mirror, it would be at 6 focal-lengths from **F,** and the image would be at 1/6 focal-lengths = 1″ anterior to **F,** or 5″ posterior to the mirror, and 1/6 the size of the object.

The nearest distance of an object from **F** in convex mirrors is 1 focal-length, which places it in contact with the metal surface of the mirror. The reciprocal of 1 is 1, so that the image is also at this metal surface, but back of it. That is the only position in which image and object can be of the same size, but the image is virtual. The object cannot be placed on the other side of **F,** without being back of the mirror, which would prevent reflection.

In Concave Mirrors.

In concave mirrors the same rule applies, but with them real images may be formed. They also permit of the object being either anterior or posterior to **F,** which is a focal-length of the mirror anterior to it, or in the direction of the object. Figure 7 illustrates the application of the rule to a concave mirror of 12″ radius, and therefore of a focal-length of 6″.

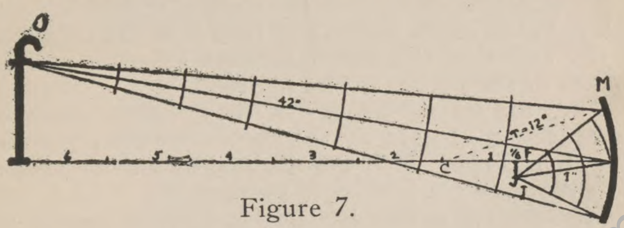

Figure 7.

In this figure, **F** is a focal-length = 6″ anterior to the mirror. If the object is 42″ anterior to the mirror, it is 36″ = 6 focal-lengths of the mirror anterior to **F.** The image is therefore, according to the rule, 1/6 of a focal-length anterior to **F,** or 1″ anterior to it, which places it at 7″ from the mirror. It is a real image and inverted, and its size is 1/6 of the dimensions of the object. This is a substantial result, as the image may be placed upon a screen.

It is obvious, with the above mirror, that the positions of image and object might be reversed. That is, an object 7″ from the mirror would be 1/6 of a focal-length anterior to **F,** and its image would then be at the reciprocal of 1/6 or 6 focal-lengths anterior to **F.** It would be a real image and 6 diameters of the object. This gives an enlarged real image of the object reflected.

If the object were placed at 4″ from the above mirror, that position is 2″ = 1/3 focal-length posterior

to **F.** The image would then be at 3 focal-lengths posterior to **F,** or 12″ posterior to the mirror. It would be an enlarged virtual image of the object. A concave mirror is unable to produce any but enlarged virtual images, and the nearer the object to **F** the greater the enlargement.

Optometric Uses.

There is but one use of the convex mirror in optometry, as far as we know, and that is in the reflection of the mires of the ophthalmometer by the convex surface of the cornea. The figures seen by the operator are the virtual images of these mires, made by the reflection of light by the surface of the cornea of the patient's eye. They are doubled by a double prism and enlarged by a telescopic system.

The concave mirror has two important optometric uses, as a retinoscopic and as an ophthalmascopic mirror. In either the mirror is relatively small and is provided with a central perforation or sight hole. The construction of the retinoscope is the same as that of a plane mirror, and mounted in the same manner. It focuses the reflected light on its way to the patient's eye and therefore gives a motion of the reflex the reverse of that of the plane mirror. Its special advantage is that it may be used for indirect ophthalmoscopy with a lens.

The ophthalmoscope is an instrument designed to afford a view of the eye ground or fundus, and to search the media forward of the retina for abnormalities or opacities. Its mirror is usually concave, so as to afford as clear an illumination of the fundus as necessary for observation through the perforation of the mirror. It is also provided with discs of small but strong lenses that can be placed between the observed and observing eyes, to correct refractive errors.

III.

OPTOMETRIC LENSES.

The thin disc of glass of which a lense is composed is transparent. It therefore permits the waves of light to pass through it, only temporarily retarding propagation while they are in the glass. It does not, like mirrors, reverse the frontages of the waves. Hence, such momentary retardation impresses the opposite of the lens curvature upon waves passing through it, the amount being according to the dioptric power or value of the lens.

Figure 8 illustrates the action of a +6 D. spherical lens upon a pencil of light from a point 20" anterior to the lens. Incident waves reach the lens with a metric curvature of +2c because of the distance of the point. The +6 lens impresses —6 on these incident waves, which therefore emerge from the lens

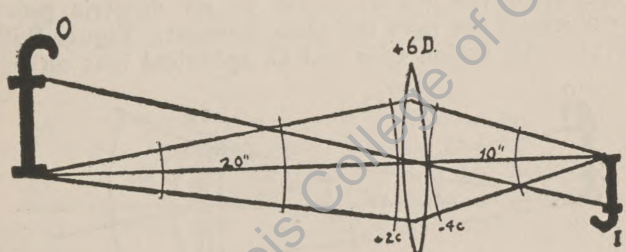

Figure 8.

with a metric curvature of —4c, and focus at 1/4 meter or 10". An object at **O** would therefore have a real and inverted image at **I.**

This is figuring the action of the lens by wave theory. In another way we may say that, since the object is at 20″, it will take +2 D. of the lens power to parallel the divergent rays from **O.** This leaves +4 D. of the lens power to act upon these paralleled rays, and it will focus such parallel rays at 1/4 meter = 10″. In some respects the latter method is simpler than the former one, but they cannot disagree in results. The real image is to the object as 10″ to 20″ = 1/2 diameter.

The principle is this, a convex or positive lens impress concavity or minus on the wave-fronts equal to its dioptric value; or a convex or positive lens converges rays of light. That is, it takes out divergence, if there is divergence, for the amount of it or the amount of its power, and with its balance of power converges the rays. By the later method we consider the power of the lens in relation to standard light, or light consisting of parallel rays.

The action of a minus spherical lens is the opposite of that of a plus lens. It impresses plus metric curvature upon the waves equal to its dioptric power, or diverges the rays the same amount. Figure 9 illustrates the action of a —3 D. spherical lens on light

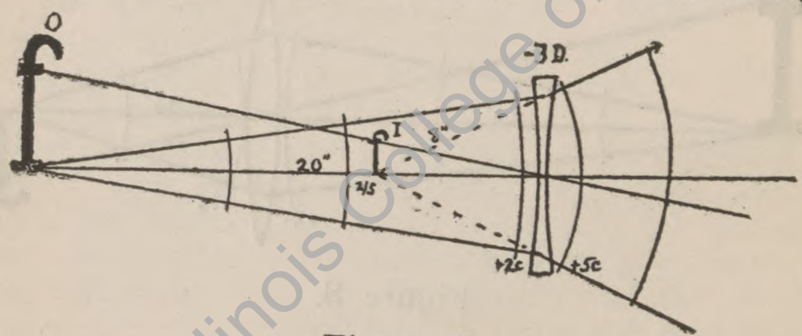

Figure 9.

from a point 20″ anterior to it. Incident waves, because of the distance of the object, are +2c. The —3

lens impresses +3c upon them, so that they emerge from it as +5c waves, or focus at 8″ anterior to the lens.

By the ray system, since a +2 lens would be required to parallel the rays, they are already divergent 2 D. The action of the —3 D. diverging lens increases this divergence 3 D. so that they emerge with a divergence of 5 D. and focus negatively at 8″ anterior to the lens. In either case the image is to the object as 8″ to 20″ = 2/5, and it is a virtual image. Other distances of the object would be calculated in the same manner, but the action of the lens would be the same for all positions of the object.

There are two surfaces to a lens, both of which act, and necessarily one a moment before the other. This allows the light to develop slightly between the two surfaces. In these thin lenses no account is taken of this factor, but the two surface actions are treated as though taking place simultaneously. In thick lenses, and in lenses of special forms, this cannot be done with accuracy.

A cylindrical lens acts in its meridian of power, at right angles to the axis, the same as a spherical lens. The axis is of **O** power, and intermediate meridians vary in power between these principal meridians, as well as from point to point. A prismatic lens deviates light in one direction only, toward the base of the prism. If such deviation is 1% of the distance, as 1 centimeter in a meter, it has a power of 1 prism-diopter. For greater deviation it has correspondingly higher power.

Dioptric Values.

The dioptric value of a lens depends upon two factors, to-wit: (1) Its optical density in excess of the medium surrounding it; and (2) the form of its

polished geometric surfaces, whether plane, spherical or cylindrical. The plano lens is the standard of dioptric value. Its dioptric value is **O,** but it is the base value, all lenses of less value being minus, all lenses of higher power being plus. It corresponds to the standard or emmetropic eye, all eyes of less refraction being hyperopic, and eyes of greater refraction being myopic.

In Figure 10 a lens is shown in which the radius of curvature of the anterior surface is 10″ and of its posterior surface is 8″. As both are convex, the anterior surface has a metric curvature of 40/10 = +4c, and the posterior surface 40/8 = +5c. The combined metric curvature of the two surfaces is there-

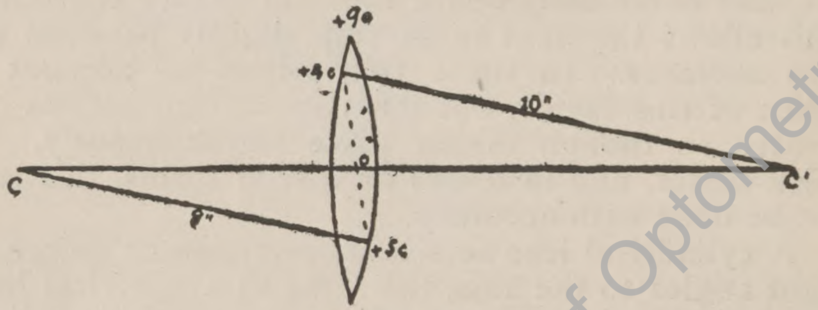

Figure 10.

fore +9c. If these curvatures were apportioned differently, as +6c anterior and +3c posterior, or +12c anterior and —3c posterior, the combined metric curvature would still be the same or +9c.

If the index of the glass of which the lens is made were 1.6, its density in excess of air would be .6. Therefore, the dioptric value of the lens would be .6 of +9 = +5.40 D. The rule of dioptric values for lenses is therefore a simple one:

Rule: Multiply the combined metric curvature of its surfaces by the excess density of the glass.

If the combined metric curvature of the surfaces is minus, the lens is also minus, but the rule is the same.

Since the dioptric value of a lens is the product of two factors, its dioptric value, divided by either of the factors, will give the other factor. This is shown for the above lens in the following simple divisions:

1. 9)5.40 = .6, the excess density, or
2. .6)5.40 = 9, the combined metric curvature.

As a +5.4 D. lens, this lens will act for that amount on light passing through it. It will impress —5.4c on all transmitted light waves, or focus parallel rays at 40/5.4 = 7.4″ about.

Algebraic Formulas.

The full expression of the action and value of lenses is best considered through algebraic formulas, in which a distinct letter or character is made the symbol of each distinct value. The symbols we will use in these formulas are as follows:

D = the dioptric value of the lens.
f = the focal-length of the lens, in meters.
C = the combined metric curvature of the surfaces.
r = the radius of curvature of a surface, in meters.
n = the optical density of the glass, compared with air.
x = the density of the glass in excess of air.
u = the distance of the object in meters, from the lens.
v = the distance of the image in meters, from the lens.

Primary Formulas.

Certain of these formulas are primary, derived directly from the factors according to the principles that have been stated. It is to be understood that unity or 1 is to be regarded, if representing distance, as 1 meter = 40"; but if representing density, as the density of air, which is the standard of density, or 1.00.

$$\mathbf{D} = 1/\mathbf{f}$$
$$\mathbf{f} = 1/\mathbf{D}$$
$$\mathbf{C} = 1/\mathbf{r}$$
$$\mathbf{r} = 1/\mathbf{C}$$
$$\mathbf{n} = 1 + \mathbf{x}$$
$$\mathbf{x} = \mathbf{n} - 1$$

In the first four formulas, if the other factor is in inches, 1 is 40".

From the direct application of what has already been expressed in regard to the value of a lens, it is obvious that

$$1.\quad \mathbf{D} = \mathbf{Cx}.$$

This is merely a condensed statement of the fact that the dioptric value of a lens is equal to its metric curvature multiplied by the excess resistence of the glass. But we derive from it the following:

$$2.\quad \mathbf{C} = \mathbf{D}/\mathbf{x}, \text{ and}$$
$$3.\quad \mathbf{x} = \mathbf{D}/\mathbf{C}.$$

In the first formula, however, since $\mathbf{C} = 1/\mathbf{r}$, by substituting $1/\mathbf{r}$ for $\mathbf{C}$, we obtain

$$4.\quad \mathbf{D} = \mathbf{x}/\mathbf{r}, \text{ and}$$
$$5.\quad \mathbf{f} = \mathbf{r}/\mathbf{x}.$$

That is, according to formula 4, if the excess density of the glass be divided by the radius, it will give the dioptric value of the surface having such radius; and

according to formula 5, if the radius is divided by the excess density of the glass, it will give the focal-length of the lens.

But there are still further formulas to be derived from these. A formula consisting of two factors and a product may always be varied so as to give an expression for either factor, as for instance:

Derived from formula 4 we have,

6. $\mathbf{x} = \mathbf{Dr}$, and
7. $\mathbf{r} = \mathbf{x}/\mathbf{D}$.

Derived from formula 5 we have,

8. $\mathbf{r} = \mathbf{fx}$, and
9. $\mathbf{x} = \mathbf{r}/\mathbf{f}$.

Object and Image.

In caluculating the action of lenses, and their effects in producing images, we use the symbols for distance of object and image from the lens. If **u** represents the distance of the object in meters, $1/\mathbf{u}$ is the metric curvature of incident waves of light; and if **v** represents the distance of the image, $1/\mathbf{v}$ is the metric curvature of the emergent waves. As the lens has impressed the opposite of its dioptric value on the waves, it follows that

10. $1/\mathbf{u} - \mathbf{D} = 1/\mathbf{v}$, or
11. $1/\mathbf{u} - 1/\mathbf{f} = 1/\mathbf{v}$.

Whether $1/\mathbf{v}$ in the above is a plus or minus value depends upon the relative values of $1/\mathbf{u}$ and $1/\mathbf{f}$. As $1/\mathbf{u}$ is a naturally positive value, if $1/\mathbf{f}$ is a negative value, its substraction from $1/\mathbf{u}$ will make $1/\mathbf{v}$ of greater positive value than $1/\mathbf{u}$. But, if **D**, and therefore $1/\mathbf{f}$, is a positive value, its substraction from $1/\mathbf{u}$ may merely reduce the positive value of $1/\mathbf{u}$, may exactly overcome it, or it may give $1/\mathbf{v}$ a minus value. The latter is necessary to make a real image.

Whatever the results, the ratio between image and object are not disturbed, but depend upon the ratio of **v** to **u.** That is, the proportion is as follows:

$$12.\quad \mathbf{I} : \mathbf{O} : : \mathbf{v} : \mathbf{u}.$$

Unless the student is familiar with algebraic formulas, no attention need to be paid to the above. He can determine results by purely numerical values alone.

By formula 7, since $\mathbf{r} = \mathbf{x}/\mathbf{D}$, an easy method of determining the radius of a tool for grinding a particular dioptric value on glass of any index is provided. For instance, with glass of 1.52, if the tool for grinding a 2.5 D. value is required, the radius is .52/2.5 = .208, or 208 millimeters. As tools are made on milling machines with a micrometer gauge, it could at once be set to this radius.

IV.

FOCAL-LENGTH SYSTEM

The focal-length system applies to lenses, the same as to mirrors, but with important differences, due to the fact that the lens has two fields, anterior and posterior, whereas the mirror has but an anterior field. A lens has two principal foci, instead of one, and distances are measured from both. Each principal focus is a focal-length of the lens from it, one anterior and the other posterior.

Figure 11 represents a +8 D. spherical lens. If a point of light is located at **F** on its principal axis, 5″ forward of the lens, divergent rays from it will be paralleled by the action of the lens. **F** is therefore its first principal focus. If parallel rays come to the

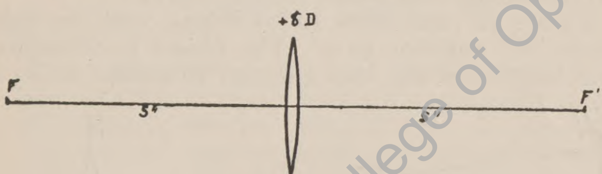

Figure 11.

lens, they will be focused at **F′**, 5″ posterior to the lens. **F′** is therefore its second principal focus. Each of the principal foci is conjugate to infinity in either direction from the lens, but neither is conjugate to the other.

The first principal focus **F** is the datum for measuring the distance of the object, while **F′** is the datum for measuring the distance of the image. Both

distances, by the focal-length system, are measured in focal-lengths of the lens. The principles that apply are as follows:

1. Object and image are in opposite directions from **F** and **F′**.
2. Whatever the distance of **O** from **F**, the image is the reciprocal of that distance from **F′**.
3. The ratio of these distances to unity represents the relative sizes of image and object.

The object may occupy a position anterior to **F**, at **F** or posterior to **F**; but the latter cannot be more than 1 focal-length of the lens in distance. If the object is anterior to **F**, the image will be posterior to **F′**; if the object is posterior to **F**, the image will be anterior to **F′**. If the distance of the object from **F** is 5 focal-lengths of the lens, the distance of the image will be 1/5 of the focal-length from **F′**. In the latter case the dimensions of the image will be 1/5 of corresponding dimensions of the object.

Figure 12 represents the +8 lens, with an object located 30″ anterior to it. The object is therefore 6 focal-lengths of the lens anterior to it, but it is only

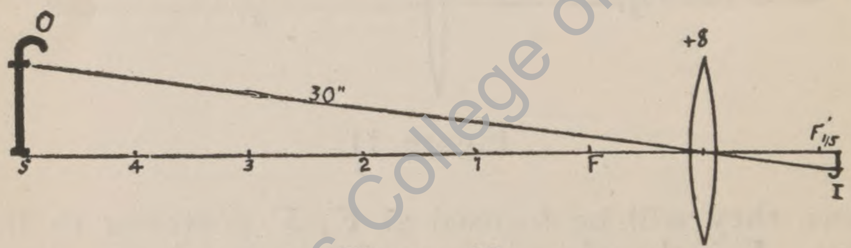

Figure 12.

5 focal-lengths anterior to **F**. In that case the image will be 1/5 of a focal-length posterior to **F′**, or 6″ posterior to the lens. It will be a real image and 1/5 diameter of the object. It is obvious therefore that the object is 5 diameters of the image.

If the above object were placed 6″ posterior to the lens, it would be 1/5 of a focal-length posterior to **F′**, in which case a real image of 5 diameters of it would be formed 5 focal-lengths anterior to **F**, or 30″ anterior to the lens. The rule of focal-lengths works both ways, and for either direction of the object from the lens. It would apply quite as well to a +5 lens, with a focal-length of 8″, or to any other positive lens whatever.

If the object, in the lens last described, were placed 4″ forward of the lens, it would be at 1/5 of a focal-length posterior to **F.** In that position the image would be located 5 focal-lengths anterior to **F′**, or 20″ forward of the lens. It would be a virtual and erect image, and 5 diameters of the object. A plus lens therefore makes real images of the same, less or greater size than the object, and virtual images of greater size than the object.

In Minus Lenses.

The same rule as to focal-lengths applies to a minus lens, but in minus lenses **F** and **F′** reverse positions. That is, the 1st principal focus **F,** is posterior to the lens, or opposite to the direction of the object. The distance of the object from **F** is therefore measured through the lens to **F,** which is 1 focal-length on the opposite side; while the distance of the image is measured from the second principal focus **F′,** which is in the same direction from the lens as the object.

Figure 13 represents the action of a —10 lens on an object situated 16″ forward of the lens. In this position it is 5 focal-lengths anterior to **F,** which is 4″ posterior to the lens. The image is therefore situated at 1/5 of a focal-length posterior to **F′**, or 4/5 of an inch posterior to it. It is 3.2″ forward of the lens.

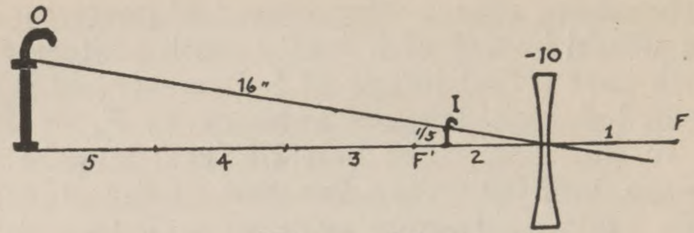

Figure 13.

The image is virtual, and 1/5 diameter of the object. It can only be seen by looking through the lens. Naturally it will look much smaller than the object, because it is smaller, although a little nearer.

This method of calculating the action and effects of lenses in producing images is important as a time saver. It enables one to solve quickly some of those problems in optics given by an optometry board. It also has an extremely practical value in photography, and in projectile lantern work of any kind.

Practical Cases.

In photo-gravure, suppose it is desired to obtain a ¼ size negative of an india ink drawing, or of an object in art or mechanics to be made into a half-tone or etching. If the camera lens has a 5″ focus, the object must be 4 focal-lengths of the lens anterior to **F.** In such a lens **F** is not 5″ from the surface, but from the nodal point, which may be an inch or more back of the surface. It can be found by focusing the camera on a distant object.

The sensitized plate, in that case, will require to be at ¼ of a focal-length from **F′,** but this can be obtained by experimenting on the ground glass. The fixing of the distance of the object is all it is necessary to do with accuracy, for the desired reduction,

for the plate will have to be put in the correct place for a perfect definition, and that is determined by trial.

In projection work, suppose that with a lens of 5″ focus it is desired to make a 40 diameter enlargement of a film, what space will be necessary? For that enlargement, the screen will have to be at a distance of 40 focal-lengths of the lens from **F′**, while the film will have to be 1/40 of a focal-length anterior to **F.** Not counting the space between the nodal points, the film and screen are 40 + 2.025 = 42.025 f.l. apart, or 210.125 inches = 17½ feet. Allowing 2 ft. for electrical wiring and condensing lights, this makes 19½ feet over all.

In this case as in the other, the distance from **F′** to the screen is the important distance. With the screen in that position, the film must be in due position to make the definition sharp at the screen. Practically, it would only be necessary to approximate these distances, for it is not likely that an exact enlargement of 40 diameters is essential, but only approximately that amount.

Optical Problems.

Suppose the applicant for a state license is given the question: With a +8 lens, where must the object be placed to produce an image 1/9 the dimensions of the object? Under the usual methods, without algebra, this might prove a puzzling problem. But, under the focal-length system, how easy it becomes. The object must be 9 + 1 = 10 focal-lengths from the lens, or at 10 × 5″ = 50″.

Suppose the question were this: In a space of 72″ between object and image, I desire to obtain an image that is 1/5 of a diameter of the object; what lens must I use and where must it stand? Let the student

work this example without algebra and by the ordinary methods. Then let him apply the focal-length system to it, and see how smoothly it gives him a $+4$ lens at 60″ from the object.

The details of the solution are as follows: In order to get this ratio of sizes between object and image, the object must be 5 focal-lengths anterior to **F**, and the image 1/5 focal-length posterior to **F′**. As **F** and **F′** are 2 focal-lengths apart, there are $5 + 2 + .2 = 7.2$ focal-lengths between them. As this distance is 72″, 1 focal-length is $72/7.2 = 10''$. Hence the lens is $+4$. The object is 6 focal-lengths $= 60''$ distant; the image is 1.2 focal-lengths $= 12''$.

It might be that a state board would ask this: With a $+5$ lens and a 16″ space, how can a real image be made of 3 diameters of the object? The applicant who understands the focal-length system will at once see that this is impossible. The space must consist of $3 + 2 + 1/3$ focal-lengths, or 5 1/3 focal-lengths in all. The space of 16″ divided this way gives 3″ for each focal-length. But a $+5$ lens has an 8″ focus, so how can the requisite number of focal-lengths be got in the space? To do so would require a space of practically 43″.

But it can readily be determined what value the lens requires to fulfill the other conditions of the object. It must be a lens with a focal-length of 3″, instead of one with a 5″ focus. The lens would therefore require to be of $+13.33+$ power. It is a good deal like asking: At 5c each how many apples can be bought with 20c at 4c apiece? It is over supplied with conditions. There is an occasional slip of this kind in state board questions, but they are not intentional.

General Proof.

The general proof of the focal-length system, as to the location of object and image, is made algebraically as follows:

Let **a** = number of focal-lengths from object to **F.**

Then **a** + 1 = number of focal-lengths from object to lens.

$\therefore$ (**a**+1) **f** = distance in meters from object to lens.

Hence,

$$\frac{1}{(\mathbf{a}+1)\mathbf{f}} = \text{metric curvature of waves at incidence.}$$

Then,

$$\frac{1}{(\mathbf{a}+1)\mathbf{f}} - \frac{1}{\mathbf{f}} = \frac{-\mathbf{a}}{(\mathbf{a}+1)\mathbf{f}} = \text{emergent curvature of waves.}$$

The focal equivalent of last is $\frac{(\mathbf{a}+1)\mathbf{f}}{\mathbf{a}} = \mathbf{f} + \mathbf{f}/\mathbf{a}$

Subtracting **f** from the last leaves **f/a,** or 1/**a** of **f.** Hence, if **a** is the focal distance anterior to **F,** 1/**a** is the focal distance posterior to **F′.**

In applying this general result to problems, it must be borne in mind that — anterior is posterior, and that the signs preceding values are signs of operation. But this applies to all algebraic formulas. If the lens is negative, its focal-length is also negative.

V.

CLASSIFICATION.

Optometric lenses are of many varieties, but they may be classified into three primary divisions: simples, compounds and specials. A simple lens is one that has but one optical element in it, as a sphere, a cylinder or a prism. A compound has two or more of these elements, on opposite surfaces. The specials are those that possess, beside the above features, some peculiarity of form, color or composition.

A spherical lens is of uniform power in all meridians, or directions across its face. It may be convex or plus (+), or concave or minus (—). The convex or plus spherical lenses are of three varieties, to-wit: plano-convex, bi-convex and meniscus. A meniscus lens is one that is convex on one surface and concave on the other. If the convex value predominates, it is a convex meniscus; if the concave value predominates, it is a concave meniscus.

Meniscus Lenses.

Lenses of this form are important, as they are the style most in use for eye glasses and spectacles. Their shape conforms to the contour of the eye before which they stand, and light from all directions passes more directly through them than lenses of a flatter form. They give, on this account, a wider field of vision to the eyes before which they stand. But meniscus lenses are more or less emphatically meniscus, as appears from the following classification.

Meniscus lenses are of three varieties, shallow, periscopic and deep. A shallow meniscus is one whose opposite curve is less than 1.25 D. That is, a plus

meniscus lens of this kind has a minus curve of less than 1.25 D., as of —.50 D. or —.75 D. If the opposite curve is —1.25 D. the lens is called a periscopic. Those lenses in which the base curve is more than 1.25 opposite the power of the lens are the deep menisci.

Figure 14 represents these varieties of meniscus lenses, with a value given for each surface. They are all plus menisci lenses. The deep meniscus may have a plus or minus base curve, most usually it is a +6 D. or a —6 D. But other base curves are often given

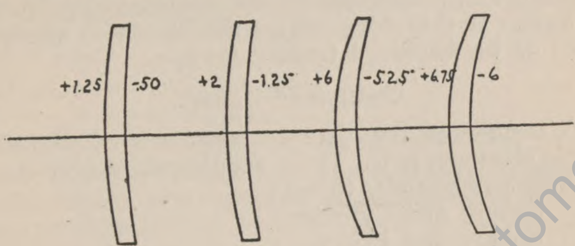

Figure 14.

them, as a +7.50 D., +9 D. or —7.50 D. or —9 D. A weaker base curve than 6 D. is also given them occasionally, but usually the standard values above are adhered to.

The minus meniscus lenses classify into the same varieties as the plus. The only difference is that, in them, the minus value predominates. A minus periscopic lens, for instance, has on it a +1.25 D. surface, and to counteract this and make the lens minus, the other surface must have a greater minus value, as —1.50 D. or more. The neutral meniscus has equal convex and concave surfaces.

The simple cylindrical lens has usually one plane surface, the opposite surface being either convex or concave and of cylindrical form. Its meridian of

greatest curvature, at right angles to its axis, is usually termed the power meridian. But a cylindrical lens has power in all meridians except its axis, which has a power of 0. These two meridians are termed together the principal meridians.

A prismatic lens, being a thin disc of glass with polished geometric surfaces (plano) is a lens. It is placed before either or both of the eyes to modify the light entering it or them. Its purpose is optical the same as that of any other lens. It deviates light equally at all points, but in one direction only, toward its base. It therefore causes the object to appear to move or be displaced toward its apex.

Compound Lenses.

A compound lens has at least two of the above simple elements in it. There are therefore four classes of these compounds, to-wit:

1. Sphere and Cylinder,
2. Sphere and Prism,
3. Cylinder and Prism,
4. Sphere, Cylinder and Prism.

In a sphero-cylinder compound, one surface is spherical and the other is cylindrical. They are the most important because the most used of the compounds.

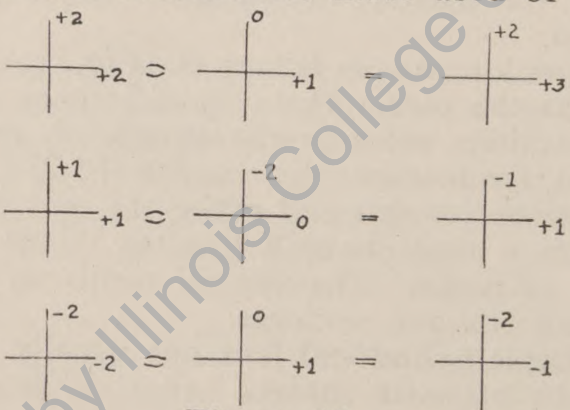

Figure 15.

In form the sphero-cylinders are of four varieties, to-wit:

1. Plus on plus,
2. Plus on minus,
3. Minus on plus,
4. Minus on minus.

The power meridian of the cylinder augments or reduces the power of the meridian of the sphere that lies under it. Hence it may neutralize or reverse it, when cylinder and sphere are of opposite sign. This gives rise to three different effects in the meridian values of a sphero-cylinder. They are as follows:

1. All +, or all meridians +.
2. All —, or all meridians —.
3. Mixed, one principal meridian +, the other —.

In case a cylinder neutralizes one meridian of the sphere, the lens is not regarded as a compound, but a simple cylindrical.

Either a spherical or cylindrical lens may have also a prismatic value, and the prismatic value can be made to extend in any desired direction across the lens. In one direction is the base of the prism, in the opposite is its apex, the line connecting them being known as the base-apex line. The base of the prism is in the direction of thickness at the edge, or greatest thickness at the edge. If possible, the prismatic value is given a lens by decentration.

It follows that a sphero-cylinder may have a prismatic value, and this is a combination of the three elements in one lens. Decentration is effective only along a meridian of power. Hence, cylindricals cannot be decentered effectively along their axis. But, a real prismatic value can be given a cylindrical lens along its axis by giving its posterior plane surface an angle to the axis. Decentration is also of slight effect in lenses of weak dioptric value.

Specials.

There is quite a list of lenses having some special feature other than those features that pertain to them generally. This list embraces principally the following varieties:

1. Toric lenses.
2. Lenticular lenses.
3. Tinted lenses.
4. Lenses of special glass.
5. Bifocal lenses.

Toric Lenses.

A toric lens is one in which one surface is given the form of a tore. That is, one principal meridian has the maximum curvature and power, while the meridian at right angles to it has the minimum curvature and power. The opposite surface of the lens may be plane or have any spherical curvature that is desired. This gives the lens a cylindrical value equal to the difference between the maximum and minimum values of the toric surface.

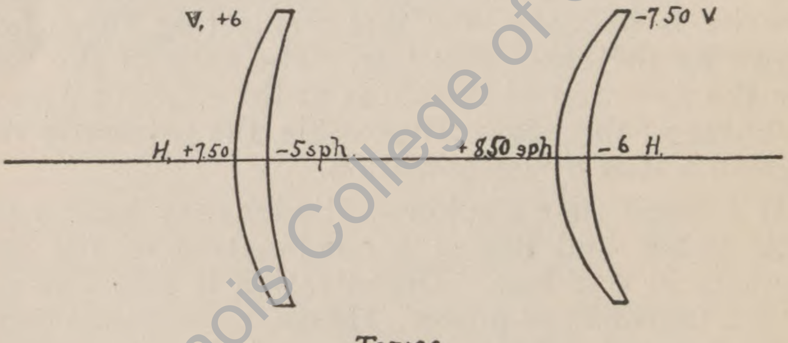

Figure 16.

The toric surface may be put upon a convex or a concave surface, the meridian of least curvature, in

either case, being termed the base curve. It is most usually +6 or —6, though other base curves are used. The trade houses call any lens of the deep meniscus variety a "toric" whether it has a cylindrical value in it or not. This is probably due to the fact that toric lenses are usually made up of the deep meniscus form. The term "peritoric" is also applied to deep meniscus sphericals, evidently to resemble the term periscopic.

Punktal Lenses.

It is well known that a spherical lens of any form has an astigmatic effect upon oblique pencils of light that pass through it. Hence, in looking at an astigmatic chart obliquely through a lens that corrects all errors, astigmatic or otherwise, the lines of an astigmatic chart to not appear equally distinct. This naturally impairs distinct vision except directly through the central portion of the lens, a space of about 30° in all, or 15° to each side of the principal axis.

The punktal lens is a lens that is specially surfaced to overcome this imperfection, and to give equally distinct vision through all areas and in any oblique direction as well as through its center. This allows the eyes to be widely rotated back of the lens without impairment of vision. The grinding of these lenses is mathematically calculated to maintain uniformity of any cylindrical value in the lens for all oblique directions. As any different correction has to be specially calculated, a pair of these lenses is quite expensive.

Lenticular Lenses.

A lenticular lens is one in which the lens value before the eye occupies a central area only of the whole lens. This may be due to a segment being ce-

mented to a base lens at the center, or to a concave depression being ground into the base lens on a central area. Minus lenticulars are usually of the latter variety, and if the base lens is a strong plus cylinder, this gives the lenticular an oblong or oval outline, its long diameter extending horizontally. The projecting margin of the base lens may be of any convenient form for mounting.

Plus lenticulars of the old style were usually made up with a small plus segment cemented to the central area of a plano lens. But more recently the segment was cemented to the concave surface of a deep meniscus lens, which might be neutral or of any value desired. The convex surface could also be made toric for the correction of astigmatism, as the cementing of a spherical lenticular to it would not neutralize the cylindrical value in the base lens. The insertion of a Kryptok segment into the lenticular would make such a lens bifocal.

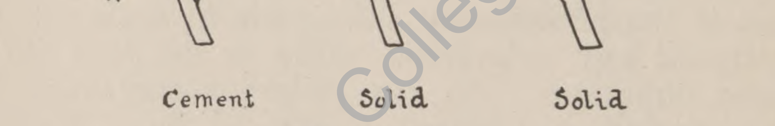

Figure 17.

More recently still these lenses, used chiefly for cataract lenses, are being ground solidly, or of one piece of glass. Some of the forms are shown in Figure 17. They make a lens of unusual lightness, even for the high powers, +9 to +16, required in these lenses.

Tinted Lenses.

The glass in any lens may be tinted, or a lens may be ground out of glass of any tint. Aside from the "smoked" lenses to be found in trial cases, which are designed to obscure the light, there is no fixed science regarding this means of improving lens values and effects. A great many amber colored lenses are prescribed, but there is considerable uncertainty as to their precise effects and value. There may be a day when this will be a better understood field, but at present no rule, even a general rule, can be laid down.

Special Glass.

There are two varieties of glass that have come into use quite recently to neutralize the harmful effects of ultra violet light. Each of them has a slight color, although it is not the color that is made the basis of claims for value. The Crookes glass, it is claimed, shuts out the ultra violet rays, and protects the eyes from their harmful effects. It has yet to be determined in just what sort of cases this glass is of greatest need and efficiency.

Noviol glass is another variety of special glass that is now used quite extensively in prescription lenses. It is claimed that this glass converts the ultra violet rays, rays that are not visible except by their chemical effects, into visual light, thereby increasing the illumination or visibility of objects. One would naturally be conservative in accepting the claims of virtue in both of these special forms of glass without some very clear demonstration of those virtues.

Bifocal Lenses.

Bifocals are lenses in which two lens areas are provided, one for distant vision and the other for near

vision, the latter area usually being the less. No field in the special construction of lenses has been so vigorously exploited as that of the bifocal-lens. There are at least ten varieties of these lenses on the market, although some of them are very similar to each other. The special names given them are as follows:

1. The split bifocal.
2. The perfection bifocal.
3. The cement bifocal.
4. The Opifex bifocal.
5. The C. P. or center perfect bifocal.
6. The solid bifocal (old style).
7. The Bisight, of one piece.
8. The Ultex, or one-piece bifocal.
9. The Kryptok or invisible bifocal.
10. The Steadfast bifocal.

It would be too "elementary" to go into a detailed description of these different forms of bifocals. They represent mechanical ingenuity rather than scientific properties,

Sizes and Styles.

There are a variety of lens sizes and styles, the regular 1, 0, 00, 000, 0000 and jumbo sizes, both in rimless and inserts; and the same in long and short ovals, as well as the round, the leaf-shaped and some shapeless, or at least of nameless shapes. The dimensions of the different sizes are to be found in the catalogues of the optical trade houses, and are hardly to be expected here.

VI.

USES OF LENSES.

The lens is of course the most important of optic agents. It is used in two principal ways: To test and measure the eyes; and to be worn, as spectacles or eye-glasses, for the correction of errors of refraction or to supplement some muscular deficiency of the eyes causing strain. The first of these uses is but a use for the purpose of examination; the latter is the final or ultimate use and purpose, and lenses of this kind we designate prescription lenses.

Trial Case.

The test lenses are usually contained in a trial case, which consists of a number of plus and minus sphericals, in pairs; a lesser number of plus and minus cylinders, also in pairs; a number of prisms arranged according to value or power; certain special discs, as the opaque disc, the pin-hole disc, the double prisms, the Maddox rod, etc.; the trial or test frames, adjustable to the face measurements; and often, though not necessarily, a retinoscope and ophthalmoscope.

The spherical lenses are usually bi-convex and bi-concave, and range in value from .12½ to 20.00 D. In the lower values the gradations between fractional lenses of different value are finer, the values to 1.00 D. being in gradations of .12½; but above 1.00 D. the gradations are not so fine but in .25's or .50's; and the higher the values the less fine the gradations become. There may be from 28 to 35 pairs of plus and minus sphericals, 16 to 24 pairs of plus and minus cylindricals, with a corresponding number of prism values.

The mounting of these lenses in large rotating discs to facilitate placing them before the eye; or the mounting of the small lenses of an ophthalmoscope in a smaller disc for the same purpose or to afford a better view of the fundus or eye ground, does not alter the character of their use as test lenses. Any arrangement of lenses in a series in the construction of an instrument for the same purpose comes under the same head. They become mere tools for a purpose.

Prescription Lenses.

These are of quite another class. They are not of any stereotyped form, but may be varied to suit the taste or purpose. For instance, a person requiring a correction of +2 D. sph., may have that value put up in any of the forms shown in Figure 18, showing cross sections of the various forms arranged along a common principal axis. **A** is the form of the trial case

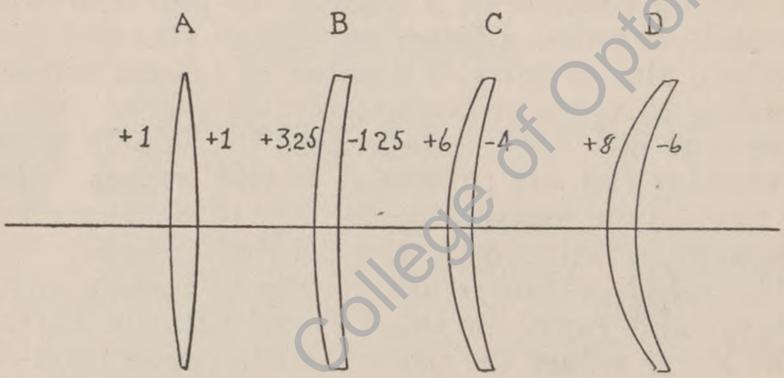

Figure 18.

lens of that value. **B** is the periscopic form. **C** is a deep meniscus lens of the same value, with a +6 base; and **D** is a deep meniscus lens with a —6 base. Minus lenses have the same variety of form.

Plus Sphericals.

Plus spherical lenses in any of these forms are used as prescription lenses to correct hypermetropia or presbyopia or both. The measure of the defect or accommodative insufficiency is made with the trial or test lenses; but the correction is supplied as a prescription lens. The value that is required for either purpose is determined by test, either subjectively or objectively, and in that work the test lenses are used; but the correction, when determined, is supplied by a prescription lens or lenses.

Young hyperopes are usually able to see distance distinctly by using the required amount of their abundant accommodation. Hyperopia that is so corrected is termed "facultative." But, as years advance, the accommodation is no longer equal to the task, and distant vision is impaired. When hyperopia impairs distant vision it is termed "absolute." In either case plus sphericals should be prescribed, the former to relieve accommodative strain, the latter to correct distant vision as well.

In the correction of hyperopia with plus sphericals, the strongest plus that does not impair distant vision is prescribed. But, in correcting presbyopia, or insufficiency of the accommodation for near vision, it is advisable to make as weak an addition to the distant correction as possible for that purpose—an addition that will still require the accommodation to act for what it can comfortably. The addition is always plus, and whatever the value of the addition, that measures the presbyopia.

Minus Sphericals.

These lenses are prescribed for myopia. In myopic eyes the plus value of the dioptric system of the

eye is too great for the depth of the eye, and light from distance is therefore focused forward of the retina, placing a diffused or blurred image of distant objects upon it. A minus spherical of the correct value neutralizes a part of this excessive plus power and focuses the parallel rays at the retina, making vision normal.

Care must be taken that the minus lens prescribed is not too strong for this purpose, for that would only result in stimulating the accommodation to neutralize part of its power, and the accommodation should be at rest for distant vision. As myopic eyes are able to see near objects with less than normal accommodation for the distance, myopes are thought not to become presbyopic as soon as hyperopes or emmetropes. With their correction for distance before the eyes, it would be discovered that they become presbyopes as early or earlier than hyperopes.

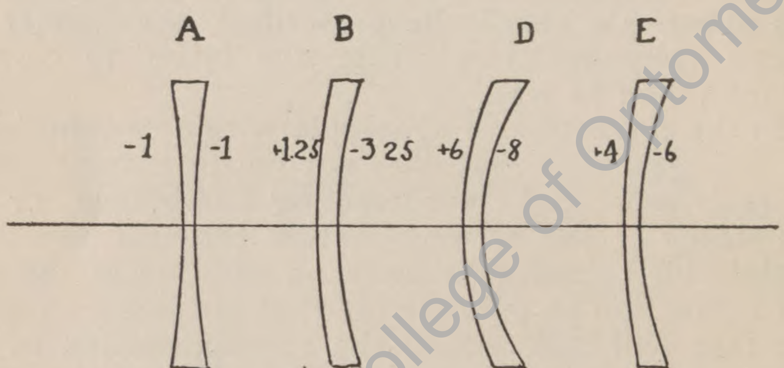

Figure 19.

Figure 19 shows a variety of lens forms that may be used to correct myopia of 2 D. **A** is the trial case form of lens. **B** is a regular periscopic lens of the same value. **D** is a deep meniscus lens of —2 D. It has a +6 base. **E** is a deep meniscus lens of —2 D. with a —6 base curve. Any plus value for the cor-

rection of presbyopi may be added to any of these lenses in the form of a cement scale. It must be surfaced to fit that surface of the lens to which it is attached, as well as provide the addition.

Cylindric Lenses.

A cylindrical lens is only prescribed for the purpose of equalizing meridians of the eye that are unequal in refraction, an optical defect known as astigmatism. The two meridians of the eye that represent the maximum and minimum refraction are, in regular astigmatism, at right angles to each other. A cylindrical lens, at the correct axis, either increases the refraction of the weaker meridian or reduces the refraction of the higher meridian, either of which equalizes the meridians, provided the cylindrical lens is of the correct power, which is determined by test.

Such inequality of refraction in different meridians of the eye results in impaired retinal images of all objects. This impairment is such that radiating lines, seen at a distance, appear of unequal darkness and distinctness. There are necessarily two focal distances in an astigmatic eye, and but one of the principal foci can be at the retina at a time. When the eye focuses at the retina horizontally, vertical lines only are clear, and vice versa for horizontal lines.

The accommodation of the eye, which is spherical, is unable to make any adjustment of the crystalline lens that will bridge this inequality, although it may put forth effort to do so. A cylindrical lens, with its axis over the normal meridian, or one that a spherical lens makes normal, has then only to be of the correct kind and value to bring the two foci together. This correction makes all of the radiating lines equally dark and distinct.

Sphero-Cylinder Lens.

As a spherical often has to be combined with a cylindrical value to completely correct the eye, the prescription lens has to be a sphero-cylinder, or a compound of sphere and cylinder. There is no such compound in the trial case. Hence, to test the error, two trial case lenses must be used together, which makes necessary two cells in the trial frame, a stationary cell for the sphere and a rotating cell for the cylinder. The latter provision allows the axis of the cylinder to be placed in any desired or required position. A half circle is provided on the trial-frame with the degree marks on it to show the position of the axis, when fixed.

A sphero-cylindrical lens that is made up in its simplest form has the spherical element on one surface and the cylindrical on the other surface. While there still are many of these compounds made up in this way, an ever increasing proportion of them are made up in the deep meniscus, and therefore toric, form. The cylindrical value has its place and effect in either form, but it is getting so that those who use the old flat forms are considered behind the times.

Prismatic Lenses.

Prisms as employed in optometry are always of a binocular character and value. They are only prescribed for those conditions that are known as muscular imbalance or heterophoria. They are used for the purpose of developing muscular conjugation of the eyes, but that is a trial case use. Prisms prescribed for permanent wear, do not, as might be supposed, correct the tendency of the eyes to deviate from normal alignment, but confirm them in that tendency.

But, they relieve the strain upon the muscles that are otherwise required constantly to act to keep the eyes in correct alignment. Any such tendency to deviate is naturally overcome by the contraction of a pair of muscles, a pair that are conjugate to each other, that rotate the eyes in opposite directions. Hence, a prism of the required value over either eye relieves the strain on both muscles—that of the eye before which it stands no more than its conjugate mate in the other eye. They are to be avoided except in imperative cases, and always by those who do not fully understand prescribing them.

Prism-Compounds.

A prismatic value may be given to a spherical lens of either variety, plus or minus; it may also be given to a simple cylindrical lens, either in its axis or meridian of power, but only in the latter by decentration. It may be given to a sphero-cylinder, and in any direction, but decentration to be effective must be on a meridian having power. A lens of this kind is required for the combination of simple errors represented or corrected by the simple lenses.

Bifocal Lenses.

Lenses of this class are prescribed only for cases that require a different value for near vision than is required for distance. In that case the correction stands for any of the following cases:

1. Hyperopia and Presbyopia.
2. Myopia and Presbyopia.
3. Astigmatism and Presbyopia.
4. Heterophoria and Presbyopia.

Even an emmetrope can use bifocals to advantage, by having a reading segment attached to his neutral

distant lenses. It saves taking off and putting on presbyopic lenses alone. The myope who needs no lenses for near work may also wear the bifocals to advantage, the reading portions being neutral, and for the same reason as the emmetrope. Special employments also may make bifocals of a special kind, or even trifocals, desirable.

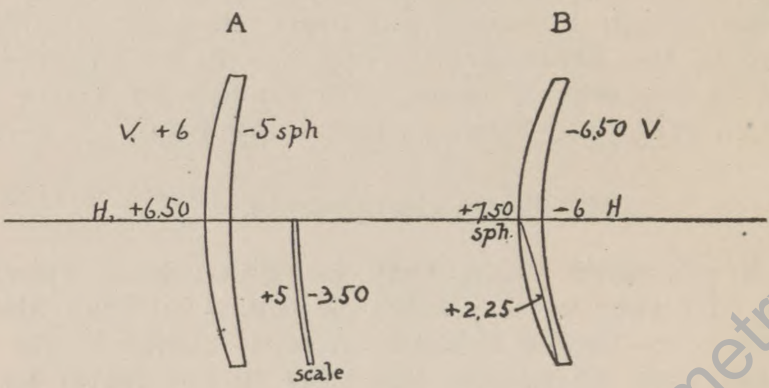

Figure 20.

Bifocals may be made up in any of the various ways referred to in the classfication of lenses (V). In toric compounds, the plus base curve, which gives a concave posterior surface, is best adapted to any variety of cement bifocals, and also to the one-piece or Ultex. But for Kryptok bifocals, the toric surface must be posterior to provide a sufficient plus spherical surface for countersinking. It therefore has the minus base curve, and the segment is in the anterior surface.

Figure 20 shows a couple of toric bifocal lenses in which either cement or Kryptok segments are used. In **A** the lens is a +1.00 sph. on +.50 cyl. made up toric on a +6 base, to which a scale of +1.50 is to be added. The surfaces of the wafer or scale are +5 on —3.50, so as to fit into the —5 of the main lens and

give the desired addition. In **B** there is a Kryptok segment. The depth of the countersinking depends upon the index of the special glass.

Assuming that **A** and **B** are for the same case, the anterior surface, for a —6 base curve, is +7.50. If the index of the special glass should be 1.60, regular glass and tools being 1.52, it would take +9.75 curvature in the segment, with the same tools, to make an addition of +1.50. As there are +7.50 on the anterior surface, the countersinking would have to be 9.75 — 7.50 = —2.25 in the main lens to provide for a +2.25 on the Kryptok segment.

The details of the calculation that gives the above results may be set down as follows:

(1)	(2)	(3)	(4)
.60	.08).520	6.5	9.75
.52	6.5	1.5	7.50
.08		9.75	2.25 = inner curve.

In (1) the density of the special glass in excess of the regular is obtained by subtraction, and is .08. In (2) we divide the factor .52 by the excess density of the special glass, which shows how much tool curvature is required for an addition of 1 D. which is, in this case, 6.50. Hence (3) to add 1.50 requires 1.5 times 6.5, or 9.75 in all on the Kryptok segment. But (4) as the front surface has +7.5, the remainder to be supplied is 9.75 less 7.50, or 2.25 D.

As Kryptok bifocals are supplied in blanks, there is no occasion to make these calculations. The blanks are made up and ready to be surfaced. But every optometrist should know the principles upon which the making of these blanks must be based.

QUIZ BOOK

Special Course No. 2

— ON —

LENSES AND MIRRORS

I.

QUIZ BOOK.

1. A fixed light is 53″ from a plane retinoscope. What is the divergence, in diopters, of the incident rays at the mirror; and what is their divergence after reflection 27″ forward of the mirror?
2. If the above incident rays have an incident angle at the mirror of 6°, at what angle from incident rays will the reflected rays leave the mirror; and how will this angle be increased 4°?
3. If a plane mirror is situated 2 meters forward of the eyes, and a reverse test chart is 2 meters from the mirror, at what distance from the eyes will the virtual image of the chart be?
4. If the distance of the mirror from the eyes is decreased 6″, and the distance of the chart from the mirror is increased the same amount, will this have any effect on the apparent distance of chart?
5. If you have a space of 10½ ft. between two walls, and locate chart and mirror on opposite walls, at what distance from the mirror must a patient be seated to make a testing space of 20 ft.?
6. A man whose eyes are emmetropic shaves himself, using a plane mirror at 16″. What accommodation will he need to use for clear vision; and what would be the effect of moving to 10″? to 8″?
7. If the above "shaver" requires +2 sphs. for reading, at which of the above distances could he use these glasses to the best advantage, and how much accommodation would he need to use with them?

8. A hyperope of 1 D. has scales of +2 on them for reading. These lenses are mounted in frames that permit them to be worn scales up or down. At what distance from the mirror could he shave without lenses?
9. In shaving himself at 20″ from the mirror, could he get along without glasses at all; and if he used the bifocals, would he place them scales up or down for shaving at 10″? at 8″? at 5″?
10. If a myope of 2 D. shaves himself before a plane mirror, what is the nearest point he can stand to see himself clearly, without accommodation? with **2** D. accommodation?

II.

1. If incident light from a point 20″ distant falls upon a convex mirror whose radius of curvature is 10″, what will be the dioptric divergence of the reflected rays?
2. Is the image formed by this action real or virtual? erect or inverted? where is it situated relative to the mirror? If the object is 10″ in height, what is the height of the image?
3. Where is the principal focus of the above mirror located? How far is the above object from **F?** How many focal-lengths of the mirror does this make? How many focal-lengths is the image from **F?**
4. If the above object is moved to a position 5″ forward of the mirror, what will be the dioptric divergence of incident rays at the mirror, and what will be the divergence of reflected rays?
5. How many focal-lengths of the mirror is the object, in the last position, from **F?** What is the reciprocal of this number? How far is the

image from **F** in focal-lengths? What is relative size of object and image in above example?

6. With what kind of a mirror can I make a real image of an object? Can such real image be made of 7 diameters of the object, and if so, how? Can it be made of 1/3 diameter of object, how?
7. With a concave mirror of 10″ radius, tell where to place an object before it so that a real image of that object, 10 times its dimensions, will be formed?
8. With the same mirror as in the last case, where must the object stand so that the mirror will make a real image of it of just ½ its actual dimensions.
9. Can an object be so placed, with respect to this mirror, as to make a virtual image of it that is 5 diameters of the object? and what position of the object will give that result?
10. If you take a position 5″ forward of the above mirror, what will incident light upon the mirror become from every point of your face? If this light comes back to your eye, what will the rays be?
11. Under the above conditions, will you see an image of yourself in the mirror? Will it appear erect or inverted? Will it appear magnified or of natural size?
12. If you approach to a position 4″ from the above mirror, what effect will this have upon incident and reflected light? If emmetropic do you require to accommodate, and how much?
13. Just how will you appear to yourself in the mirror at the last distance named, magnified or reduced? erect or inverted? natural or reversed? How can you obtain an inverted image of yourself?

III.

1. In what manner does a +9 spherical lens act upon light from a point 10″ forward of the lens; and what is its effect in producing the image of an object located at that distance?
2. If a —3 spherical lens receives light from an object 8″ forward of it, how does it act upon light from that point; and what is its effect in producing an image of the object?
3. A certain lens has +7c curvature on its anterior surface and —5c upon its posterior surface. If the index of the glass is 1.5, what is the dioptric value of the lens?
4. If the combined curvature of the two surfaces of a lens is —5c, and its anterior surface is +3c, what is the curvature of its posterior surface? For an index of 1.6 what is its dioptric value?
5. Having glass of an index of 1.585, I wish to grind a surface having a dioptric value of +6 D. In order to grind this value what must be the radius of the tool or lap used for that purpose?
6. An object is 12″ from a lens; the lens makes an image of the object that is 24″ from the lens. If the image is real, what is the dioptric value of the lens?
7. If a +5 D. surface is ground upon standard glass of an index of 1.52, and the same tool is used to surface a special glass of an index of 1.624, what will be the dioptric value of the latter?
8. If the standard glass of 1.52 is ground with a —5 D. on one surface, and a plus curve to fit it is ground on the special glass of 1.624, and the two are fitted together, what is their combined value?

9. If glass of 1.52 is ground with a —8 D. curve, and a plus curve to fit it is ground on glass of an index of 1.585, and the two are fitted together, what is their combined dioptric value?
10. If, with glass of 1.52, and surfacing tools for the same, there is combined as above a glass of an index of 1.56, what surfacing of the two will be necessary to give a combined value of +1 D.?
11. Trace the course of light from a point 20″ forward of a +6 lens, to and through the lens, and across a space of 2″ to and through a —4 lens, and state what the results will be.
12. An object is 10″ forward of a +6 lens; there is then a space of 12″, and then a lens of —3 D. After emerging from the last lens at what point will there be a focus?
14. Describe the image formed in the last example: Is it real or virtual, erect or inverted, and what is its size compared with object?

IV.

1. An object 4″ high is 50″ forward of a +10 lens. By locating its two principal foci, and calculating distances in focal-lengths from them, determine location and size of image.
2. What would be the effect, with the above lens and object, of moving the object to a distance of 110″ from the lens? What also would be the effect of moving it to a distance of 12″?
3. Suppose that with the above lens and object, it is desired to make a real image of the object 20″ in height. Where should the object be placed before the lens to get this result?

4. Suppose, that, with the same lens and object, it is desired to make a virtual image of the object at the anterior principal focus (**F**) of the lens. Where would the object be placed?
5. If it is desired to make a real image of the object that is 2 diameters of it, or 8″ in height, where would the object have to stand and where would the image be located?
6. With a +8 lens, what space would be required in which to make the lens produce an image of the object enlarged four diameters? Could such image be made either virtual or real?
7. If the lens in a camera has a focal-length of 6″, where must the object be placed before the camera in order to get a negative of it of 1/3 the diameter of the original?
8. Having a projecting lantern the lens of which has a 4″ focal-length, at what distance from the screen must the lantern stand to give an image on the screen 36 diameters of the negative?
9. With a +5 D. lens, what space is required in which to produce a real image that is 2½ diameters of an object placed before it, and in what position, relative to the lens, must the object be?
10. An object is 12″ anterior to a minus lens whose focal-length is 6″; at what distance from the lens will the image be, will it be real or virtual, and what will be its size compared with object?
11. An object is 20″ anterior to a +4 lens, 12″ from the +4 lens is a —5 lens. If light from the object passes through the two lenses, at what distance from the latter will the image be?
12. An object and a screen are separated by a space of 62½″. In that space it is desired to produce a real image that is 4 diameters of the object. What lens is necessary and where will it be placed?

13. Two lenses, a +8 and —10 are separated by a space of 2". If an object is placed 10" forward of the +8, where will the image be formed; is it real or virtual, and what is its size compared with object?

V.

1. A spherical lens has +1.25 on its anterior surface, and —.50 on its posterior surface. What is the dioptric value and to which class of spherical lenses does it belong?
2. If a +1.00 scale were to be cemented to the concave surface of above lens, what would be the surfacing of the scale, and to what class of spherical lenses would the scale belong?
3. Tell how a lens of the same value (+.75) would be made periscopic in form; and how would a +1.00 scale be surfaced for cementing to it, and to what class would the scale belong?
4. A patient requires —.75 sph. for distance, with +1.50 added for near vision. How would this lens be made up periscopic, and what would be the surfacing of the scale for cement bifocals?
5. The correction of an eye is +1.50 sph. on +.75 cyl. ax. 90; and the addition required for reading is +2.00 sph. What is the prescription for a reading lens only for above case?
6. If the above correction were made up toric, on a +6 base, what would the posterior surface of the distance glass measure with the lens measure, and what would be horizontal power of front surface?
7. What surfacing of a scale to make the +2 addition required would be necessary for the dis-

tance correction in this form? and to what class of spherical lenses would the scale belong?

8. If the correction of an eye is —1.50 cyl. ax. 180 for distance, and the addition for reading is +1.50 sph., what would the simplest form of reading lens be?
9. If above distance correction is made up toric on a +6 base, what is the dioptric curvature of the posterior surface, as shown by the lens measure, and of the meridians of the front surface?
10. How would a +1.50 scale for bifocals to be cemented to the above lens be surfaced, and to which class of spherical lenses would the scale made in that way belong?
11. The distance Rx for an eye is +.75 —1.25 ax. 180. To which class of sphero-cylinders does this lens belong, both as to its form and its value?
12. Made up toric on a —6 base, what is the spherical curve on the anterior surface, as measured with the lens measure, and what is its value on the other surface, by same means?
13. If this lense is 1.52 glass, what countersinking of 1.624 glass in its front surface would make a +1.50 Kryptok segment or addition to the distance Rx?

VI.

1. The prescription for a lens is +1.25 sph. —.75 cyl. x 180. Transpose this prescription to one in which a +.75 cyl. is used in place of one of —.75 x 180.
2. Which of the above would be best adapted to a +1.50 cement scale, and how would such scale be surfaced for attachment to the main lens, and how would it be mounted before the eye?

3. Reduce the above lens to the toric form, stating the value and position of each different curve; and how would a +1.50 scale be surfaced for attachment to this lens?

4. If the same lens were made up toric on a —6 base curve, what curvature, in diopters, would the lens measure show for front surface, and for each meridian of the toric surface?

5. If the regular glass had an index of 1.5232, with tools for the same, and a Kryptok segment of glass whose index is 1.5886 were in it, to add +1.50, what would be the inner curve?

6. If a Kryptok bifocal, the main lens and segment having exactly the same curvatures as the last, gave an addition of + 2.00, of what index is the special glass of which the segment is made?

FINAL EXAMINATION.

With the completion of the work given in the preceding pages, the student will be ready for his Final Quiz or Examination. This will consist of twenty to twenty-five new questions, embracing problems on any of the various principles and actions contained in this booklet.

A standing of 100 per cent is expected on all of the foregoing Quizzes, and on the Final Examination.

Optometry
and
Humanity

Extracts from an address delivered before Convention of Optical Society of the State of New York held at Rochester, New York, June 4—5, 1917.

The Practice of Optometry

"The practice of Optometry is now regulated by law in over 40 States of the Union and is legally defined 'as the employment of any means other than the use of drugs for the measurement of the powers of vision and the adaptation of lenses for the aid thereof.'"

Message to the Public

"What is the real message we have for the public? The message is revealed in the blessings that follow when defects of vision are relieved, weak vision made strong and the eye sight of advancing age kept up to the standards of youth."

In the Relief of Humanity

"Consider for a moment what optometry has done and is doing for the vast body of men, women and children whose eyes are myopic. To them new fields of vision are opened through the administrations of the optometrist. Until the proper, scientifically prescribed glasses are applied, the myope's world is fogged and restricted. He does not see as man should see the full beauties of color, form and harmony in the created world. After the optometrist applies the proper glasses, there comes a wonderful change to the myope, who then begins to see a new world with all its marvels."

INCREASING MEN'S EFFICIENCY

"People suffering from hyperopic and astigmatic defects of vision are readily relieved in our day, so that their vision is made normal and more comfortable and free from eye strain. Their efficiency in their daily labors as students and workers is increased. They are enabled to compete upon even terms with their fellows in life's struggle. They can perform tasks that would otherwise be impossible; they can perform an amount of labor under which otherwise they would break down from nervous and physical ailments. Thousands of men and women depend upon their ability to use their eyes at close work to make a living. In many, many instances, eye strain would drive them away from their occupation were it not for the aid that is given to their vision by means of the lenses applied after the optometrist makes a scientific examination of their eyes."

PRESBYOPIA NO LONGER DREADED

"In the old days, many workers broke down early, because of failing eyesight and eyestrain. Men and women of middle age, because of their failing accomodation, found themselves hampered in doing close work with their eyes. In studying, writing and numerous industrial occupations, involving the use of machinery, tools, etc., at close range, thousands of workers would lose

their means of livelihood were it not for the services of the optometrist in aiding vision and relieving eyestrain. Presbyopia is no longer a terror to be dreaded. The man of advancing age, wearing the proper glasses, may read and write with the same ease as in his younger days and in many cases better. The woman of advancing age may attend to her sewing and household tasks with equal ease."

Widespread Blessings of Optometry

"In every field of activity the blessings of optometry are apparent. In the home, the school, the office, the church, the workshop, the amusement halls, the beneficial influence of the optometrist's work is felt. Literature, science and art share in this benefit. The workers in each of these lines of effort obtain better vision as a result of the optometrist's science and are thus enabled to put forth greater and more successful effort. Add to all this, the fact that the proper correction of defective vision conserves nervous energy and thus tends to prevent various nervous and physical ailments, and physical improvement often means moral and intellectual improvement—and then we realize more fully the immense importance and good of our profession to humanity."